

11/26 Recitation

1. In class we showed that the SVD rank k approximation of a matrix A gives the best rank k approximation of A when measured using the Frobenius norm. It turns out that this is also true if we use the *spectral norm*. The spectral norm of a matrix M , denoted $\|M\|_S$, is the largest singular value of M . Consider the following SVD of a matrix A . Let

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 2 \\ -3 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{5} & 0 & 0 & 2/\sqrt{5} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -2/\sqrt{5} & 0 & 0 & 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} 2\sqrt{5} & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{bmatrix}^T$$

- (a) Find the SVD rank 2 approximation A_2 of A .
- (b) Calculate the Frobenius and spectral distances between A and A_2 ; i.e. compute $\|A - A_2\|_F$ and $\|A - A_2\|_S$ (Feel free to use Julia). Which distance is smaller?
- (c) Create some random matrices A in Julia (using the “randn(A)” command) and compute their Frobenius and spectral norms. Can you say anything in general about their relationship?

2. Visit <https://timbaumann.info/svd-image-compression-demo/> and play around with the demo. It provides various images of different sizes and provides compressed images using SVD rank k approximations that you can compare using a slider. For each image, for what value of k is the image fairly clear? What does this say about the singular values σ_i , where $i > k$?
3. Work through the **Julia14PCA** notebook for a PCA data visualization example.